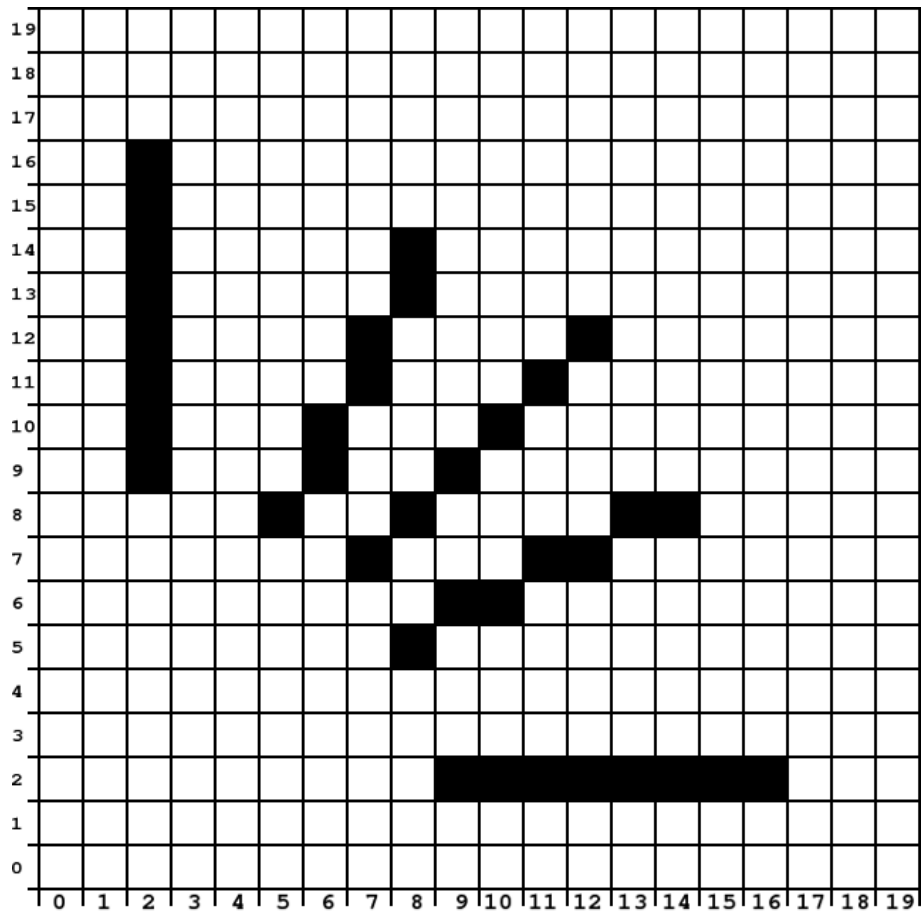


Drawing Lines with Pixels

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Summary

Computers draw lines and circles during many common tasks, such as using an image editor. But how does a computer know which pixels to darken to make a line?

Skills

- 2D coordinates
- Using an algorithm
- Geometry

Materials

- Pen or pencil
- Worksheet
- Ruler and compass (to check lines and circles after drawing them)

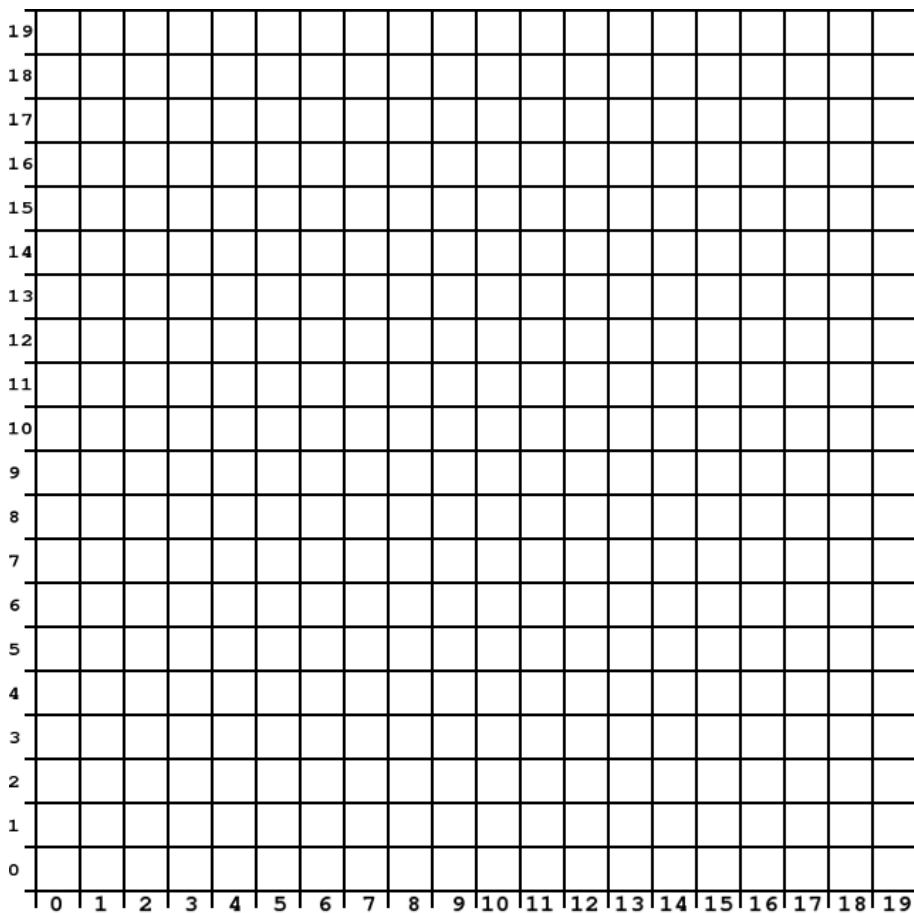
1 Some easy lines

Computers draw images using pixels. Pixels are the tiny squares that make up the image you see on computer monitors. If you look carefully at a computer screen with a magnifying glass, you can see the individual pixels.

To draw a line, a computer must work out which pixels need to be filled so that the line looks straight. We can try this by colouring in squares on a grid.

On the following grid, try to draw these straight lines by filling in pixels in the grid:

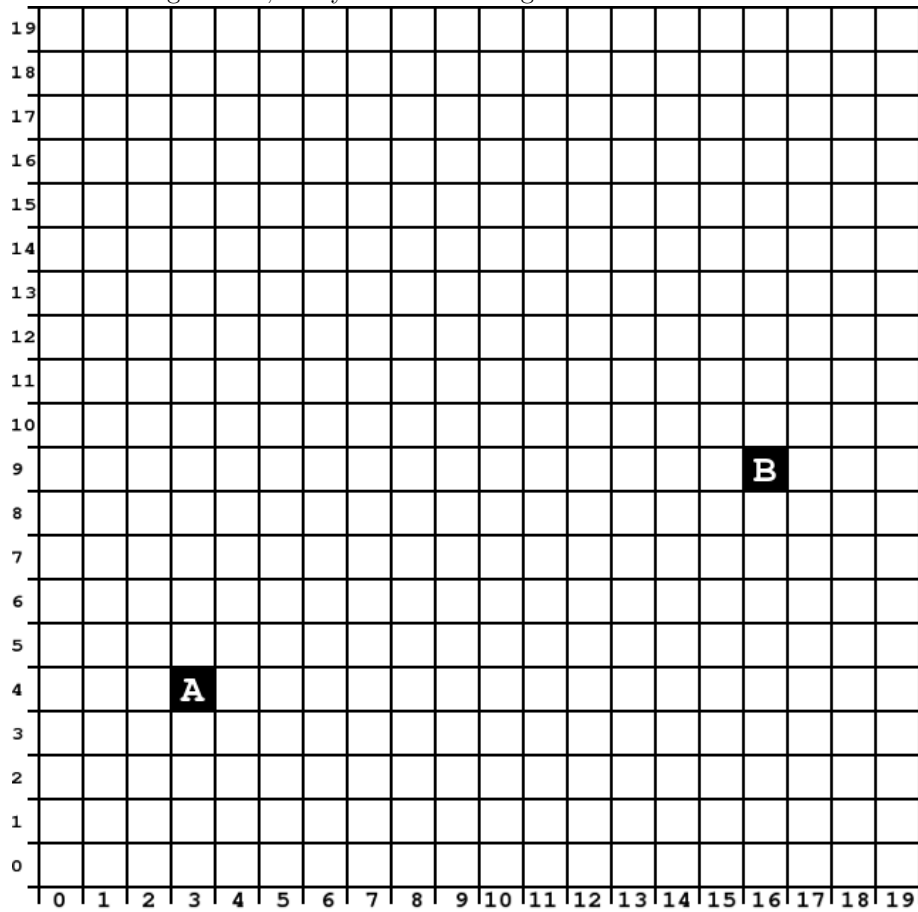
- $(2, 17) \rightarrow (10, 17)$
- $(18, 2) \rightarrow (18, 14)$
- $(1, 5) \rightarrow (8, 12)$



Check with a ruler. Are your lines straight?

2 A more difficult line

Without using a ruler, can you draw a straight line from A to B?



Once you have finished drawing your line, try checking it with a ruler. Place the ruler so that it goes from the centre of A to the centre of B. Does it cross all of the pixels that you have coloured?

3 Bresenham's Line Algorithm

One way that a computer can know which pixels to colour, is with Bresenham's Line Algorithm. It follows these simple rules:

$A = 2 \times \text{change in Y value}$

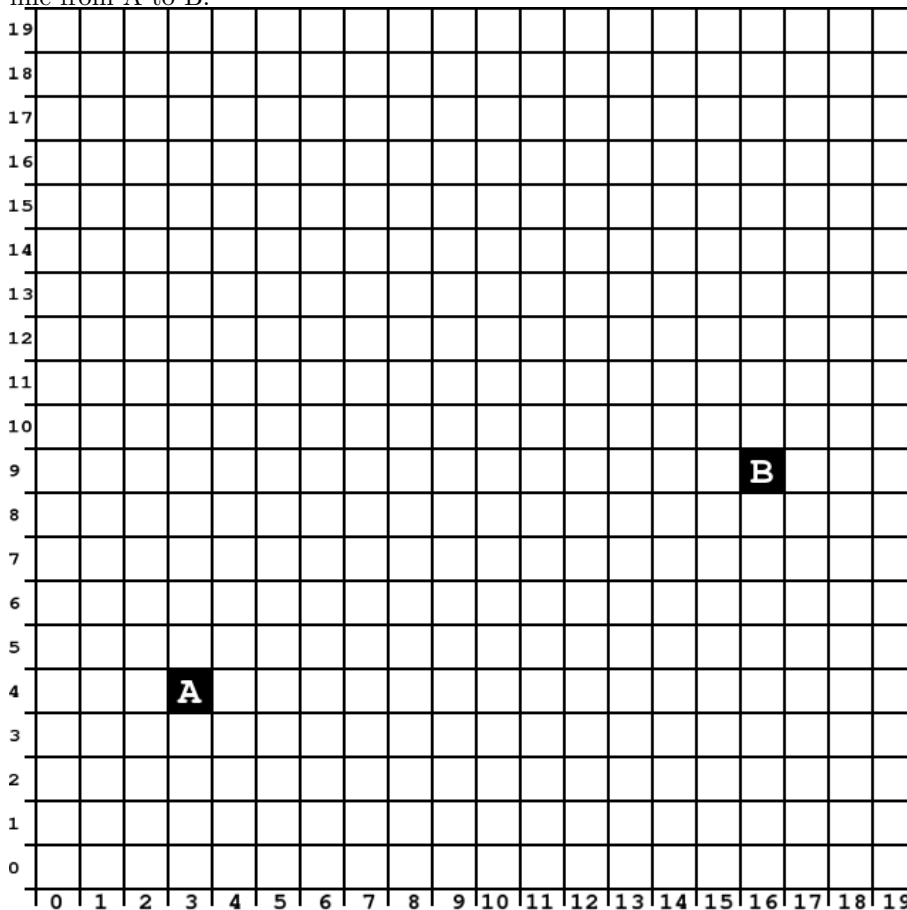
$B = A - 2 \times \text{change in X value}$

$P = A - \text{change in X value}$

Fill the starting pixel. Then for every position along the X axis:

- if P is less than 0, draw the new pixel on the same line as the last pixel, and add A to P .
- if P was 0 or greater, draw the new pixel one line higher than the last pixel, and add B to P .
- continue this process until we reach the end of the line.

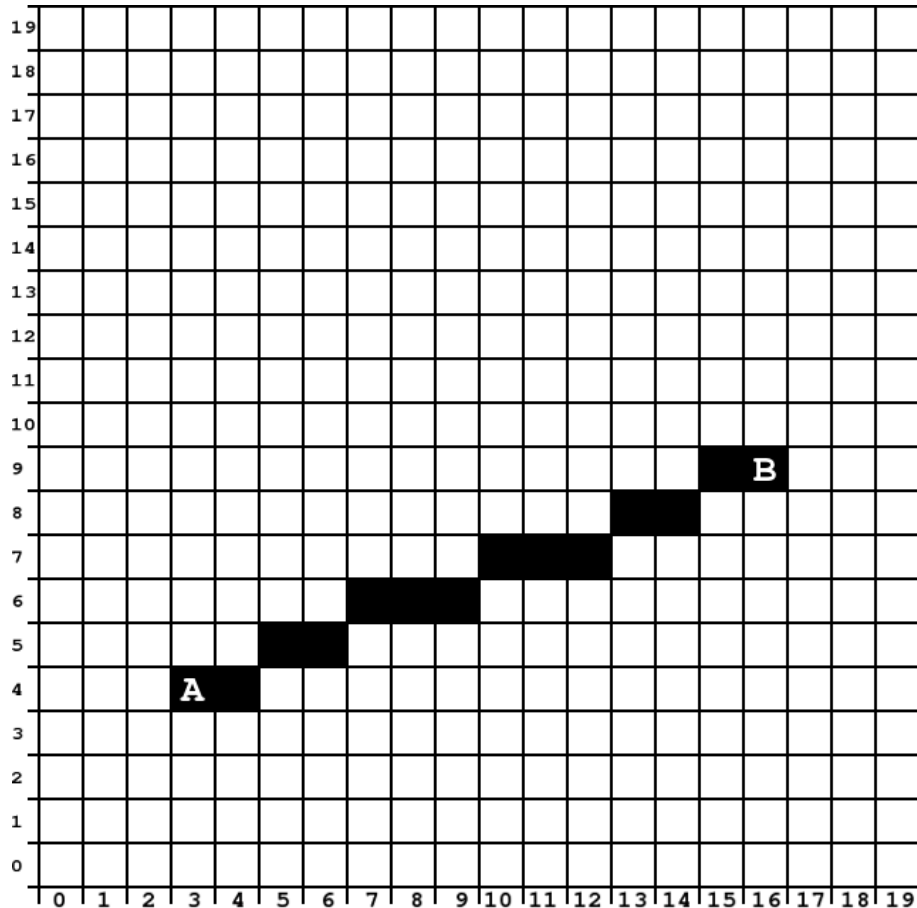
Without using a ruler, use Bresenham's Line Algorithm to draw a straight line from A to B:



Once you have completed the line, check it with a ruler. How does it compare to your first attempt?

Teacher's answer sheet for the line A→B

Correct pixels



Correct values during calculation

$$A = 10, B = -16$$

$P_0 = -3$, Draw the next pixel along on the same row as the starting pixel.

$P_1 = 7$, Draw the next pixel along on the row above the previous pixel.

$P_1 = -9$, Draw the next pixel along on the same row as the previous pixel.

$P_3 = 1$, Draw the next pixel along on the row above the previous pixel.

$P_4 = -15$, Draw the next pixel along on the same row as the previous pixel.

$P_5 = -5$, Draw the next pixel along on the same row as the previous pixel.

$P_6 = 5$, Draw the next pixel along on the row above the previous pixel.

$P_7 = -11$, Draw the next pixel along on the same row as the previous pixel.

$P_8 = -1$, Draw the next pixel along on the same row as the previous pixel.

$P_9 = 9$, Draw the next pixel along on the row above the previous pixel.

$P_{10} = -7$, Draw the next pixel along on the same row as the previous pixel.

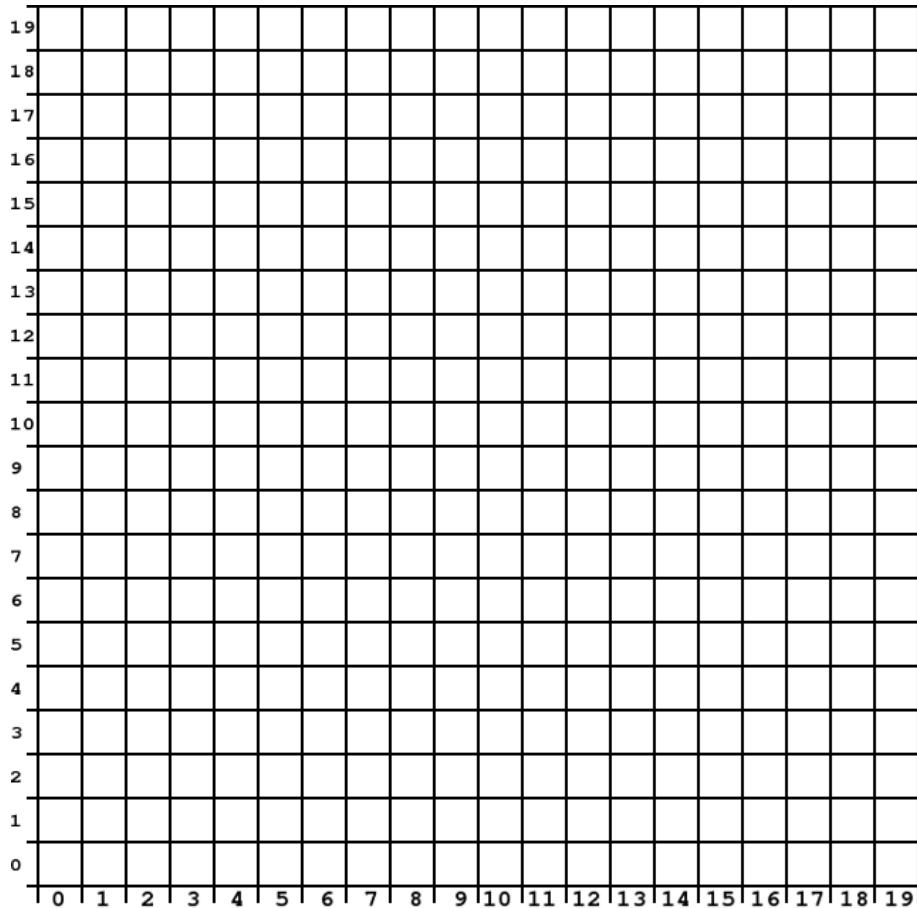
$P_{11} = 3$, Draw the next pixel along on the row above the previous pixel.

$P_{12} = -13$, Draw the next pixel along on the same row as the previous pixel.

4 Lines on other angles

So far, the version of Bresenham's line drawing algorithm that you have used only works for lines that have a gradient (slope) between 0 and 1. To make this algorithm more general, so that it can be used to draw any line, some additional rules are needed:

- If a line is sloping downward instead of sloping upward, then when P is 0 or greater, draw the next column's pixel one row below the previous pixel, instead of above it.
- If the change in Y value is greater than the change in X value, then the calculations for A , B , and the initial value for P will need to be changed. When calculating A , B , and the initial P , use X where you previously would have used Y , and vice versa. When drawing pixels, instead of going across every column in the X axis, go through every row in the Y axis, drawing one pixel per row.



In the grid above, draw your own personal line. For the start point of the line, choose the X value as the first letter in your first name converted to a number (Eg, Caren would be 3, because C is the third letter in the alphabet). If the number is greater than 19, subtract 20 from it. For the Y value of the start point, choose the number made with the second letter of your first name.

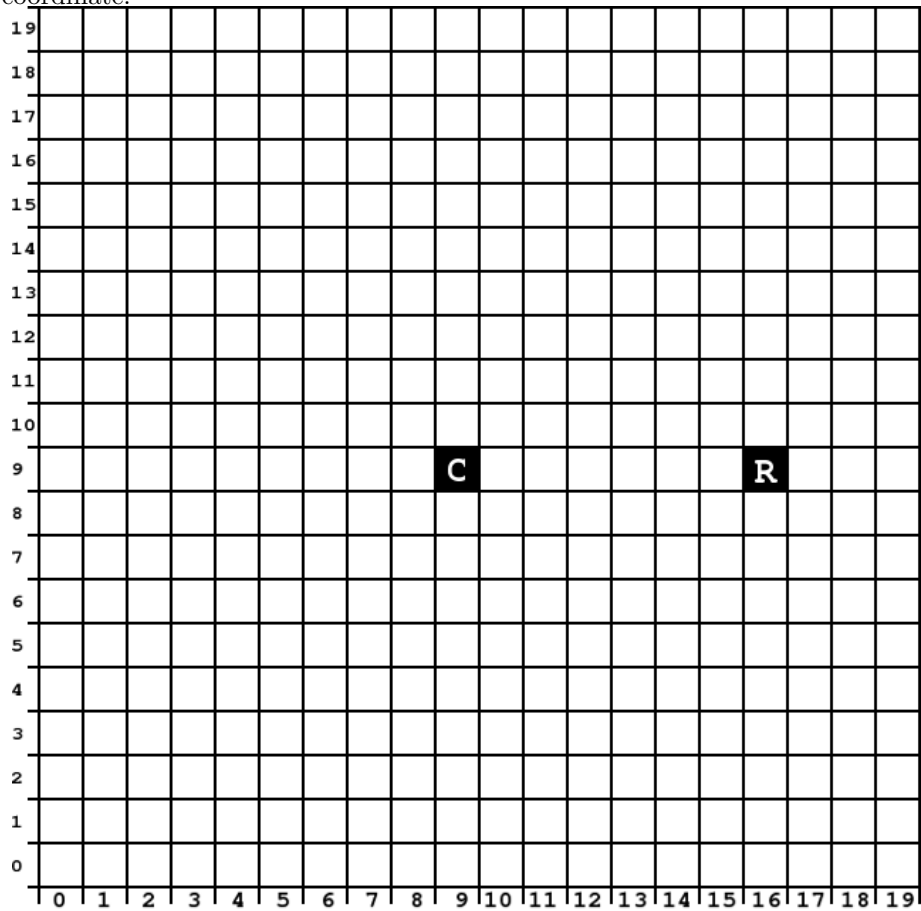
For the end point of the line, use the first and second letters of your last name. For example: If your name was John Smith, you would use the 'Jo' in John to choose the starting point (10, 15). You would then use the 'Sm' in Smith to choose the ending point (19, 13).

Use Bresenham's algorithm to draw the line.

5 Circles

As well as straight lines, another common shape that computers often need to draw are circles. An algorithm similar to Bresenham's line drawing algorithm, called the Midpoint Circle Algorithm, has been developed for drawing a circle efficiently.

A circle is defined by a centre point, and a radius. Points on a circle are all the radius distance from the centre of the circle. The centre point has an X and Y coordinate.



Try to draw a circle by hand by filling in pixels (without using a ruler or compass). Note how difficult it is to make the circle look round.

6 Midpoint Circle Algorithm

Here are the rules for the Midpoint Circle Algorithm:

$E = -\text{Radius}$

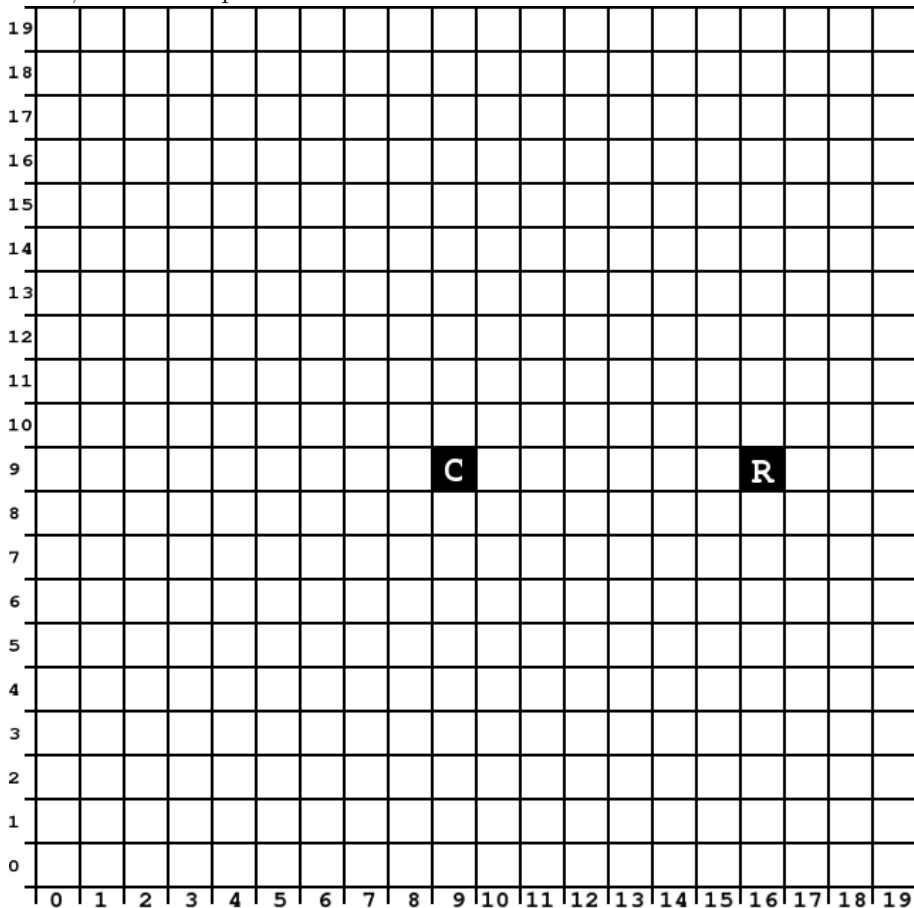
$X = \text{Radius}$

$Y = 0$

Until Y becomes greater than X , repeat the following rules in order:

- Fill the pixel at coordinate $(X + \text{Centre}_X, Y + \text{Centre}_Y)$
- Increase E by $2 \times Y + 1$
- Increase Y by 1
- If E is greater than or equal to 0, subtract $(2X - 1)$ from E , and then subtract 1 from X .

Follow the rules to draw a circle on the grid, using C as the centre of the circle, and R as a point at radius distance from the centre:



When Y becomes greater than X , one eighth (an octant) of the circle is drawn. The remainder of the circle can be drawn by reflecting the octant that

you already have. Reflect pixels along the X and Y axis, such that the line of reflection crosses the middle of the centre pixel of the circle. Half of the circle is now drawn, the left and the right half. To add the remainder of the circle, another line of reflection must be used. Can you work out which line of reflection is needed to complete the circle? Think about the reflection required before turning the page to check the answer.

To complete the circle, you need to reflect along the diagonal. The line of reflection should have a gradient of 1 or -1, and should cross through the middle of the centre pixel of the circle.

While using a line of reflection on the octant is easier for a human to understand, a computer can draw all of the reflected points at the same time it draws a point in the first octant.

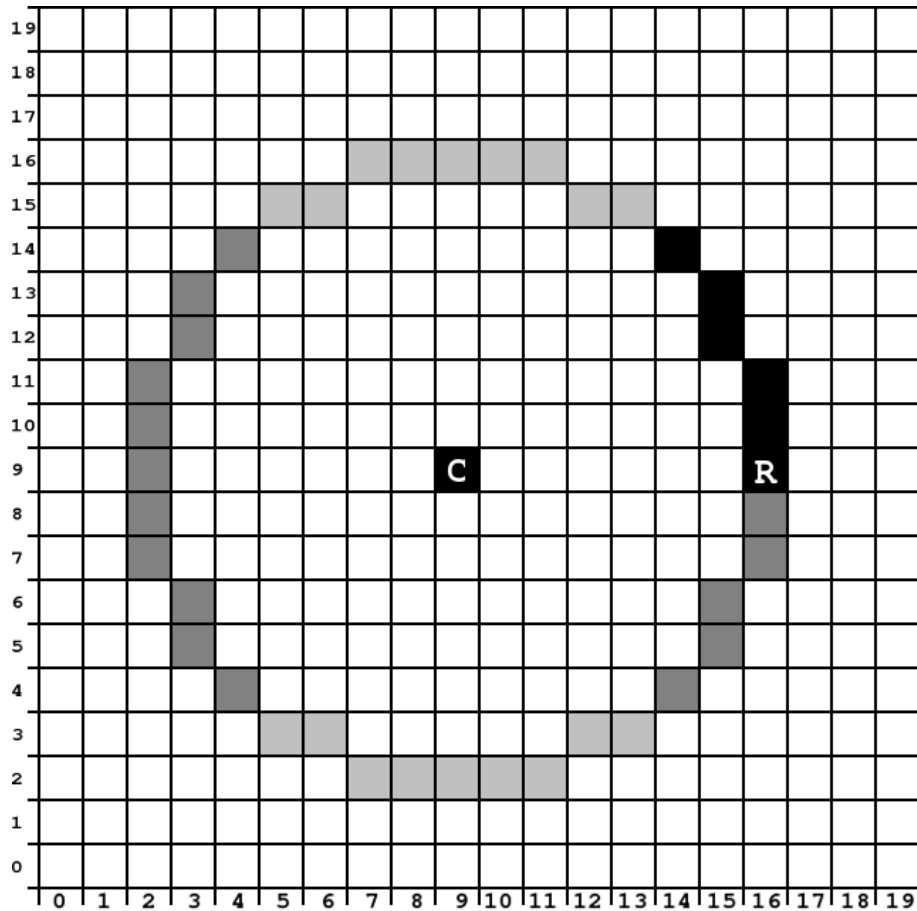
7 What's it all about?

Computers need to draw lines and circles for a wide variety of tasks. From game graphics to lines in an architect's drawing and even to a tiny circle for the dot on the top of the letter 'i' in a word processor. By combining line and circle drawing with filling and antialiasing, computers can draw smooth, clear images. When an image on a computer is described as an outline and fill colours it is called vector graphics. Vector graphics can be re-drawn at any resolution. This means that with a vector image, zooming into the image will not cause the pixelation seen when zooming in to bitmap graphics.

Outline fonts are one of the most common uses for vector graphics. Allowing the text size to be increased to very large sizes, with no loss of quality to the letter shapes.

Teacher's answer sheet for the Midpoint Circle Algorithm

Correct pixels



The black pixels represent the initial octant of the circle drawn by the algorithm. The darker gray pixels represent reflection along the X and Y axis. The lighter gray pixels represent the reflection along a diagonal.

Correct values during calculation

$E_0 = -7, X_0 = 7, Y_0 = 0$; Plot pixel (16, 9).

$E_1 = -6, Y_1 = 1$; Plot pixel (16, 10).

$E_2 = -3, Y_2 = 2$; Plot pixel (16, 11).

$E_3 = 2, Y_3 = 3$;

$E_4 = -11, X_4 = 6$; Plot pixel (15, 12);

$E_5 = -4, Y_5 = 4$; Plot pixel (15, 13);

$E_6 = 5, Y_6 = 5$;

$E_7 = -6, X_7 = 5$; Plot pixel (14, 14);

$E_8 = 5, Y_8 = 6$; Y is greater than X, so we can now reflect our octet.

